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On Q- Finitisticness of Fuzzy Bitopological spaces

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Abstract: In this paper we have introduced the concept of Q- Finitisticness and pair wise Q- Finitisticness of Fuzzy Bitopological spaces and studied its various properties. In the end we tried to generalize the results for L-Bitopological spaces also.

Keywords: *Bitopological space; Finitisticness; L- topology; Open refinement.* ***AMS 2000 Mathematics Subject Classification:*** *54A40, 04A72.*

Introduction and Preliminaries

In the year 1965, an American Mathematician L**. A. Zadeh**[17] introduced the concept of Fuzzy Set and studied its various properties. Thus a new type of set theory took birth, which is known as fuzzy set theory. This historic paper of Zadeh enthralled mathematicians all over the world and they started to study almost all the mathematical concepts based on Cantors set theory in terms of fuzzy set theory. In this way a new branch of mathematics started to emerge which is today known as fuzzy mathematics. In 1968, a Chinese mathematician, **C. L. Chang**[5] introduced the concept of fuzzy topology and developed basic notions for these spaces. After this paper [5] of Chang, many papers throughout the world were published on this subject. **Goguen** [7,8] generalized the concepts of fuzzy set and fuzzy topology and introduced the concepts of L-fuzzy set and L-fuzzy topology in the years 1967 and 1973 respectively.**Hohle-Rodabaug**[9] used the terms **L-topology** and **L-topological Space** for the terms L-fuzzy topology and L-fuzzy topological space. The first book on fuzzy topology appeared in 1997 by Chinese mathematicians **Liu-Luo**[12].

The concept of finitistic space in general topology was introduced in 1960 by **R. G. Swan**[16]. Of course Swan did not name these spaces as finitistic. The term "Finitistic" was used by **Bredon** in his book in 1972. In general topology the study of finitistic spaces became interesting with the basic paper of **Deo-Tripathi** in which they proved a Characterization Theorem for non-finitistic paracompact spaces. **Jamwal-Shakeel** introduced the concept of Finitistic

Space in Fuzzy an L-Fuzzy topology and proved its several properties. Myself continued the study of these spaces introduced the several versions of finitistic spaces by using

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different open covers, filters, Ideals and Grills. The concept of **Bitopological Space** was introduced by **Kelly**[11] in 1963.

The **order**[14] of a family $\{U_\lambda:\lambda\in\Delta\}$ of subsets, not all empty, of some set X is the largest integer n for which there exists a subset M of Δ with n+1 elements such that $\cap_{\lambda\in M}U_\lambda$ is nonempty, or is ∞ if there is no such largest integer.

Let $\Delta \neq \varnothing$ and $A = \{A_\lambda:\lambda\in \Delta\}$ be a family of fuzzy subsets of a nonempty set X. Then **order** [10] of A is defined as under:

Case-I. When $A_\lambda\neq\underline{0}$ for atleast one value of λ in Δ. Then the order of A is the largest nonnegative integer n for which there exists a subset M of Δ having n+1 elements such that $\wedge_{\lambda\in M} A_\lambda \neq \underline{0}$ or is ∞ if there is no such largest integer n.

Case-II. When $A_\lambda = \underline{0}$ for all $\lambda\in \Delta$. Then the order of A is –1.

A **Bitopological space** is a triple (X, T_1, T_2) where X is a nonempty set and T_1, T_2 are two topologies on X. Let (X, T_1, T_2) be a Bitopological space. A subfamily $\{U_\lambda : \lambda \in \Lambda\}$ of T_i is said to be **T_i open cover** of (X, T_1, T_2) where i=1,2 if $\cup_{\lambda \in \Lambda}U_\lambda = X$. A Bitopological space (X,T_1,T_2) is said to be **Compact** if each T_i open cover of X has T_j finite sub cover where i, j = 1,2 and i ≠ j. Any function A: X → I where I = [0, 1] is called a **fuzzy subset**[17] of X. The set of all fuzzy subsets of X is denoted by I^X. A subfamily $\delta \subset I^X$ is said to be a **fuzzy topology**[10] on X if (i) $\underline{0}$ $\underline{1}\in\delta$, (ii) $\{U_\lambda : \lambda \in \Lambda\}\subset \delta \Rightarrow \vee_{\lambda \in \Lambda}U_\lambda\in\delta$, (iii) $U, V\in\delta \Rightarrow U \wedge V\in\delta$. The pair (X,δ) is called fuzzy topological space. For every $a\in I$, $\underline{a}$ is called "a" valued constant function from X to I. A POSET L is said to be a **Complete Lattice** if for all subset A of L, $\vee A \in L$ and $\wedge A \in L$. Let X be any set and L be a complete lattice. Then any function A : X → L is called L-fuzzy subset of X.

Let X be a nonempty set and L be complete lattice with an order reversing involution $':L\to L$. The operation $':L^X\to L^X$ is defined by using the order reversing involution $':L\to L$ as: For $A\in L^X$, $A'(x) = (A(x))'$, $\forall x\in X$. $':L^X\to L^X$ is called **pseudo-complementary operation** on L^X and A' is called **pseudo-complement** of A. The set of all L-fuzzy subsets of X is denoted by L^X. Let X be any set and L be a complete lattice. Then any function A : X → L is called **L-fuzzy subset** of X. The set of all L-fuzzy subsets of X is denoted by L^X. A subfamily δ of L^X where L is a complete lattice with an order reversing involution $':L \to L$ is said to be **L- topology** on X if (i) $\underline{0}$,$\underline{1}\in\delta$, (ii) $\{U_\lambda: \lambda \in \Lambda\}\subset \delta \Rightarrow \vee_{\lambda\in\Lambda}U_\lambda\in\delta$, (iii) $U, V\in\delta \Rightarrow U \wedge V\in\delta$. The pair (X,δ) is called L-topological space. The members of δ are called **L-fuzzy open subsets** of X. Let (X,δ) be a fuzzy topological Space and $A\in\delta$. Then A' is called **L-fuzzy closed subset** of X and it is defined as $A'(x) = (A(x))'$. Let (X,δ_1) and (Y,δ_2) be two fuzzy topological spaces. A function f: $X\rightarrow Y$ is said to be **continuous** from (X,δ_1) to (Y,δ_2) if $\forall$ $U \in\delta_2 \Rightarrow Uf \in\delta_1$. (Note Uf means Uof). Let (X, δ) be a fuzzy topological space. Then $[\delta] = \{U:\chi_U \in \delta\}$ is a general topology on X. The general topological space $(X, [\delta])$ is called **background space** of (X, δ). A **Fuzzy Bitopological Space** is a triple (X, δ_1, δ_2) where X is a nonempty set and δ_1, δ_2 are two fuzzy topologies on X. A function f:$(X, \delta_1, \delta_2) \rightarrow (Y, \delta_3, \delta_4)$ said to be continuous if $\forall$ $U\in\delta_3$orδ_4, $Uf\in\delta_i$ for i = 1,2.

Let A be a fuzzy subset of X. Then **support** of A is denoted by Supp(A) and it is defined as $Supp(A) = \{x\in X: A(x) > 0\}$. Let (X, δ) be a fuzzy topological space and A be any fuzzy subset of X. A subfamily μ of δ is said to be Q-open cover of A if $\forall$ $x\in$ Supp(A), there exists some $U\in\mu$ such that $A'(x)<U(x)$. μ is said to be Q-open cover of (X, δ) if it is Q-open cover of $\underline{1}$. For most of the preliminaries used in this paper, we refer reference 12 and 15. For terms on categories and functors used in this paper, we refer reference 13.

Main definitions and results

Definition2.1. Let (X,δ_1,δ_2) be a fuzzy Bitopological space. Let A be any fuzzy subset of X. A is said to be Q- Finitistic in (X,δ_1,δ_2) if every δ_i Q-open cover of A has a finite order δ_j Q- open refinement where i, j = 1,2.

Definition2.2. A Bitopological space (X, δ_1, δ_2) is said to be Q-finitistic if $\underline{1}$ is Q-finitistic in (X,δ_1,δ_2)

Definition2.3. Let (X,δ_1,δ_2) be a fuzzy Bitopological space. Let A be any fuzzy subset of X. A is said to be pair wise Q- Finitistic in (X,δ_1,δ_2) if every δ_i Q-open cover of A has a finite order δ_i Q- open refinement for all i= 1, 2.

Definition2.4. A Bitopological space (X, δ_1, δ_2) is said to be pair wise Q-finitistic if $\underline{1}$ is pair wise Q-finitistic in (X,δ_1,δ_2).

Theorem2.5. Let (X, T_1, T_2) be general Bitopological space and $A\subset X$. The A is finitistic in (X, T_1,T_2) if and only if χ_A is Q-finitistic in $(X, \chi(T_1), \chi(T_2))$ where χ is characteristic functor from **Top** to F-**Top.**

Proof. Here (X, T_1, T_2) be general Bitopological space, $A\subset X$ and $(X,\chi(T_1), \chi(T_2))$ is a fuzzy Bitopological space. Here χ_A is fuzzy subset of X. Suppose A is finitistic in (X, T_1, T_2). Let $\mu = \{\chi_{U\alpha}:\alpha\in\Delta\}$ be any $\chi(T_i)$ Q- open cover of χ_A in $(X,\chi(T_1), \chi(T_2))$. We show that $\nu = \{U\alpha:\chi_{U\alpha}\in\mu\}$ is T_i open cover of A in (X,T_1, T_2). Let $x\in A$. Then $\chi_A(x) = 1>0$. But $\chi_A(x) > 0\Rightarrow x\in Supp(\chi_A)$ and μ is $\chi(T_i)$ Q- open cover of $\chi_A \Rightarrow$ there exists some $\chi_{U\lambda x}$ in μ such that $\chi'_A(x)< \chi_{U\lambda x}(x)$. But$\chi'_A(x) < \chi_{U\lambda x}(x)\Rightarrow \chi_{U\lambda x}(x)= 1\Rightarrow x\in U_{\lambda x} \Rightarrow \nu$ is T_i open cover of A in (X,T_1, T_2). Since A is finitistic in (X, T_1, T_2), therefore ν has T_j finite order open refinement say $\nu_1 = \{V_\lambda:\lambda\in\Delta_1\}$. It can be easily verified that $\mu_1 = \{\chi_{V\lambda}: V_\lambda\in\nu_1\}$ is $\chi(T_j)$ is finite order Q- open refinement μ. Hence χ_A is Q finitistic in $(X,\chi(T_1), \chi(T_2))$.

Converse. Suppose χ_A is Q-finitistic in $(X, \chi(T_1), \chi(T_2))$, we have to show that A is finitistic in (X, T_1, T_2). Let $\mu = \{ U_\alpha: \alpha\in\Delta\}$ is T_i open cover of A in (X,T_1, T_2). We show that $\nu = \{\chi_{U\alpha}: U_\alpha\in\mu\}$ is $\chi(T_i)$ Q- open cover of χ_A in $(X,\chi(T_1), \chi(T_2))$. Let $x\in Supp(\chi_A)$. Then $\chi_A(x)>0$. But $\chi_A(x)>0\Rightarrow \chi_A(x) = 1\Rightarrow x\in A$. But $x\in A$ and $\mu = \{ U_\alpha: \alpha\in\Delta\}$ is T_i open cover of A in $(X,T_1, T_2)\Rightarrow$there exists some $U_{\lambda x}\in\mu$ such that $x\in U_{\lambda x}$. Then $\chi_{U\lambda x}\in\nu$ and $\chi'_A(x)< \chi_{U\lambda x}(x)$. Hence $\nu = \{\chi_{U\alpha}: U_\alpha\in\mu\}$ is $\chi(T_i)$ Q- open cover of χ_A in $(X,\chi(T_1), \chi(T_2))$. Since χ_A is Q-finitistic in $(X, \chi(T_1), \chi(T_2))$, therefore ν has a $\chi(T_j)$ finite order Q- open refinement say $\nu_1 = \{\chi_{V\beta}: \beta\in\Delta_1\}$. Then it can be easily checked that $\mu_1 = \{ V_\beta: \chi_{V\beta}\in\nu_1 \}$ is finite order T_j open refinement of μ. Hence A is finitistic in (X, T_1, T_2).

Theorem2.6. A general Bitopological (X, T_1, T_2) is finitistic if and only if $(X, \chi(T_1), \chi(T_2))$ is Q-finitistic**.**

Proof follows by Theorem2.5.

Theorem2.7. Let (X,δ_1,δ_2) be a fuzzy Bitopological space. Then $(X,[\delta_1],[\delta_2])$ is finitistic if and only if $(X, crs\delta_1, crs\delta_2)$ is Q-finitistic.

Theorem2.8. If (X,δ_1,δ_2) is a Q-finitistic fuzzy Bitopological space, then it is pair wise Q-finitistic.

Proof. Suppose (X, δ_1, δ_2) is a Q-finitistic fuzzy Bitopological space. We have to show that (X, δ_1, δ_2) is pair wise Q-finitistic spaces. Let $\mu = \{U_\lambda : \lambda\in\Lambda\}$ be any δ_i Q-open cover of (X, δ_1, δ_2). Since (X,δ_1,δ_2) is Q-finitistic fuzzy Bitopological space, therefore μ has a δ_j finite order Q-open refinement say ν. Again since ν is δ_j Q-open cover of (X,δ_1,δ_2) and (X,δ_1,δ_2) is Q-Finitistic, therefore ν has a δ_i finite order Q-open refinement say μ_1. The clearly μ_1 is δ_i finite order Q-open refinement of μ. Hence (X, δ_1, δ_2) is pair wise Q-finitistic spaces.

Remark 2.9. Converse is not true. See the following example:

Example2.10. Let X = {a, b} be a set having two elements. Let $\delta_1 = \{\underline{0}, \underline{1}\}$ and $\delta_2 = \{\underline{0}, \chi_a, \chi_b, \underline{1}\}$. Then (X,δ_1,δ_2) is pair wise Q-finitistic fuzzy topological spaces. But (X,δ_1,δ_2) is not Q-finitistic because $\{\chi_{\{a\}},\chi_{\{b\}}\}$ is δ_2 Q-open cover of (X,δ_1,δ_2) which has no δ_1 finite order Q-open refinement.

Theorem2.11. Let (X, δ_1, δ_2) be a weakly induced fuzzy Bitopological space and A be a fuzzy subset of X. Then A is Q-finitistic in $(X, \delta_1, \delta_2) \Rightarrow A_{(0)}$ is finitistic in $(X, [\delta_1], [\delta_2])$.

Proof. Let A be Q-finitistic in (X, δ_1, δ_2). We have to show that $A_{(0)}$ is finitistic in $X, [\delta_1], [\delta_2])$. Let $\mu = \{U_\alpha:\alpha\in\Delta\}$ be any $[\delta_i]$ open cover of $A_{(0)}$ in $(X, [\delta_1], [\delta_2])$. By definition of $(X, [\delta_1], [\delta_2])$ each $\chi_{U\alpha}$ is δ_i open fuzzy subset of X in (X, δ_1, δ_2). We show that $\nu = \{\chi_{U\alpha}: U_\alpha\in\mu\}$ is δ_i Q-open cover of A in (X, δ_1, δ_2). Let $x \in Supp(A)$. Then $A(x)>0$. But $A(x)>0 \Rightarrow x\in A_{(0)}$. Since $\mu = \{U_\alpha:\alpha\in\Delta\}$ is $[\delta_i]$ open cover of $A_{(0)}$ in $(X, [\delta_1], [\delta_2])$, there exists some $U_{\alpha x}\in\mu$ such that $x\in U_{\alpha x}$. Now $x\in U_{\alpha x}\Rightarrow\chi_{U\alpha x}(x) = 1\Rightarrow A'(x) < 1 = \chi_{U\alpha x}(x) \Rightarrow A'(x) < \chi_{U\alpha x}(x)$. Clearly $\chi_{U\alpha x}\in\nu$. Hence $\nu = \{\chi_{U\alpha}:\alpha\in\Delta\}$ is δ_i Q-open cover of A in (X, δ_1, δ_2). Since A is Q- finitistic in (X, δ_1, δ_2), therefore ν has a δ_j Q-finite order open refinement say $\nu_1 = \{W_\lambda:\lambda\in\Delta_1\}$. Now we shown that $\mu_1 = \{(W_\lambda)_{(0)}: W_\lambda\in\nu_1\}$ is finite order $[\delta_j]$ open refinement of μ. Since (X, δ_1, δ_2) be a weakly induced, therefore each $(W_\lambda)_{(0)}$ is $[\delta_j]$ open subset in $(X, [\delta_1], [\delta_2])$. Let $x\in A_{(0)}$. Then $A(x)>0$. But $A(x)>0\Rightarrow x\in supp(A)$. Since ν_1 is δ_j Q-open cover of A, there exists some $W_{\lambda x}\in\nu_1$ such that $A'(x)< W_{\lambda x}(x)$. But $A'(x)< W_{\lambda x}(x) \Rightarrow W_{\lambda x}(x)> A'(x) \geq 0 \Rightarrow W_{\lambda x}(x)> 0\Rightarrow x\in(W_{\lambda x}(x))_{(0)} \Rightarrow\mu_1$ is $[\delta_j]$ open cover of $(X, [\delta_1], [\delta_2])$. It can be checked that μ_1 refinesμ. Now we show that order of μ_1 is finite. Here order of ν_1 is finite. Let order of ν_1= m. Let $\{(W_1)_{(0)}, (W_2)_{(0)}, (W_3)_{(0)},\ldots\ldots\ldots,(W_{m+2})_{(0)}\}$ be any subfamily of μ_1 having m+2 elements. We show that $\cap^{m+2}{}_{i=1}(W_i)_{(0)} = \phi$. Suppose $\cap^{m+2}{}_{i=1}(W_i)_{(0)} \neq \phi$. Then there exists some $x\in\cap^{m+2}{}_{i=1}(W_i)_{(0)}$. But $x\in\cap^{m+2}{}_{i=1}(W_i)_{(0)}\Rightarrow x\in (W_i)_{(0)}\ \forall\ i = 1,2,3,\ldots\ldots m+2\Rightarrow W_i(x)> 0\ \forall\ i = 1,2,3,\ldots\ldots m+2\Rightarrow\wedge^{m+2}{}_{i=1}W_i\neq \underline{0} \Rightarrow$ order of ν_1 is exceeding m. This is a contradiction. Hence order of μ_1 is not exceeding m. Hence $A_{(0)}$ is finitistic in $(X, [\delta_1], [\delta_2])$.

Remark2.12. Nothing can be said regarding the converse part of the above theorem. However for pair wise Q-finitisticness we have the following result:

Theorem2.13. Let (X, δ_1, δ_2) be a weakly induced fuzzy Bitopological space and A be a fuzzy subset of X. Then A is pair wise Q-finitistic in (X, δ_1, δ_2) if and only if $A_{(0)}$ is pair wise finitistic in $(X, [\delta_1], [\delta_2])$

Theorem2.14. Let (X, T_1, T_2) be any general Bitopological space and A be any fuzzy subset of X. Then A is pair wise Q-finitistic in $(X, \omega(T_1), \omega(T_2))$ if and only if $A_{(0)}$ is pair wise finitistic in (X, T_1, T_2).

Proof. Since $(X, \omega(T))$ is induced fuzzy topological space and we know every induced fuzzy topological space is weakly induced, therefore $(X, \omega(T))$ is weakly induced fuzzy. Here $[\omega(T)] = T$. Hence by Theorem 2.13 A is pair wise Q-finitistic in $(X, \omega(T_1), \omega(T_2))$ if and only if $A_{(0)}$ is pair wise finitistic in (X, T_1, T_2).

Theorem2.15. Pair wise Q-finitisticness is good extension property of Pair wise finitisticness in general topology for Bitopological spaces.

Proof. Since $\underline{1}_{(0)} = X$. Hence by Theorem 2.14, $\underline{1}$ is pair wise Q-finitistic in $(X, \omega(T_1), \omega(T_2))$ if and only if $\underline{1}_{(0)}$ is pair wise finitistic in (X, T_1, T_2). i.e $(X, \omega(T_1), \omega(T_2))$ is pair wise Q-finitistic if and only if (X, T_1, T_2) is pair wise finitistic.

Theorem2.16. Every closed subspace of a Q- finitistic fuzzy Bitopological is Q- finitistic.

Proof. Let (X,δ_1,δ_2) is a Q-finitistic fuzzy Bitopological space and $(Y, \delta_1|_Y, \delta_1|_Y)$ is a closed subspace of (X,δ_1,δ_2). We have to show that $(Y, \delta_1|_Y ,\delta_1|_Y)$ is Q-finitistic. Let $\mu = \{U_\lambda : \lambda\in\Lambda\}$ be any $\delta_i|_Y$ Q-open cover of $(Y, \delta_1|_Y ,\delta_1|_Y)$.Then each $U_\lambda = V_\lambda|_Y$ for some $V_\lambda \in T_i$ where i, j = 1, 2 and $i\neq j$. We show that $\nu = \{V_\lambda : U_\lambda = V_\lambda|_Y \ \forall\ U_\lambda \in\mu\}\cup\{\chi_{Y'}\}$ is δ_i Q open cover of X. Let $x\in X$. Then $x\in Y$ or $x\in Y'$.

Case-I. If $x\in Y$, then there exists some $U_\lambda\in\mu$ such that $\underline{1}'(x) < U_\lambda(x)$. Then clearly

$\underline{1}'(x) < V_\lambda(x)$ because $U_\lambda = V_\lambda|_Y$ and $V_\lambda \in\{V_\lambda : U_\lambda = V_\lambda|_Y \ \forall\ U_\lambda \in\mu\}$.

Case-II. If $x\in Y'$. But $x\in Y' \Rightarrow\chi_{Y'}(x) = 1>0 = 1'(x)< \chi_{Y'}(x)$.

Hence $\nu = \{V_\lambda : U_\lambda = V_\lambda|_Y \ \forall\ U_\lambda \in\mu\}\cup\{\chi_{Y'}\}$ is δ_i Q-open cover of (X,δ_1,δ_2).

Since (X,δ_1,δ_2) is Q-finitistic, therefore ν has δ_j finite order Q-open refinement say $\nu_1 = \{W_\alpha: \alpha\in\Delta\}$. Then clearly $\mu_1 = \{W_\alpha|_Y: W_\alpha\in\nu_1\}$ is $T_j|_Y$ is finite order Q-open refinement of μ. Hence $(Y ,\delta_1|_Y, \delta_2|_Y)$ is Q-finitistic.

Theorem2.17. Every Q- compact fuzzy Bitopological is Q- finitistic.

Proof. Let (X,δ_1,δ_2) be a Q-compact fuzzy Bitopological space. We have to show that (X,δ_1,δ_2) is Q-Finitistic. Let $\mu = \{U_\lambda : \lambda\in\Lambda\}$ be any δ_i Q-open cover of X. Since (X,δ_1,δ_2) is a Q-compact, therefore μ has a δ_j finite Q-sub cover say $\{U_1, U_2, U_3, \ldots, U_n\}$.Then $\nu = \{\underline{0}, U_1, U_2, U_3, \ldots, U_n\}$is δ_j finite order Q-open refinement of μ. Hence (X,δ_1,δ_2) is Q-Finitistic.

Theorem2.18. Let (X, δ_1, δ_2) be a fuzzy Bitopological space and A be a fuzzy subset of X such that Supp(A) is finite. Then A is pair wise Q-finitistic in (X, δ_1, δ_2).

Proof. Here (X, δ_1, δ_2) is a fuzzy Bitopological space and A is a fuzzy subset of X such that supp(A) is finite. We have to show that A is pair wise Q-finitistic in(X, δ_1, δ_2). Let $\mu = \{U_\lambda : \lambda\in\Lambda\}$ be any δ_i Q-open cover of A in (X,δ_1,δ_2). Here supp(A) is finite. Suppose supp(A) = $\{x_1, x_2, x_3, \ldots, x_n\}$. Since μ is Q-open cover of A in (X,δ_1,δ_2), therefore for each $x_i\in$ supp(A), there exists some $U_{\lambda i}$ such that $A'(x_i) < U_{\lambda i}(x_i)$. Hence $\nu = \{\underline{0}, U_{\lambda 1}, U_{\lambda 2}, U_{\lambda 3}, \ldots, U_{\lambda n}\}$is finite order δ_i Q-open refinement of μ. Hence A is pair wise Q-finitistic in(X, δ_1, δ_2).

Remark In Theorem2.18, A need not be Q-finitistic. See the following example:

Let X = N(The set of all natural number). Define $A:X\to I$ as A(1), A(2) are non-zero where A(n) = 0 $\forall n\in N-\{0,1\}$. Then A is fuzzy subset of X and supp(A) = $\{1,2\}$. Let $\delta_1 = \{\underline{0}, \underline{1}\}$ and $\delta_2 = \{\underline{0}, f, g, \underline{1}\}$ where f, $g:X\to I$ defined as $f(1) = g(2) =1$ where $f(n) = 0$ $\forall n\in N-\{1\}$ and $f(n) = 0$ $\forall n\in N-\{2\}$. Then clearly (X, δ_1, δ_2) is a fuzzy Bitopological space. Here A is not Q-finitistic because the δ_2 Q-open cover $\{f, g\}$ of A has no δ_1 Q-open refinement.

Theorem2.20. Let A and B be two Q-finitistic fuzzy subsets in (X, δ_1, δ_2). Then $A\vee B$ is also Q-finitistic in (X, δ_1, δ_2).

Proof. Let $\mu = \{U_\lambda : \lambda\in\Lambda\}$ is δ_i Q-open cover of $A\vee B$ in (X,δ_1,δ_2). Then μ δ_i Q-open cover of A as well as B. Since both A and B are Q-finitistic fuzzy subsets in (X, δ_1, δ_2), therefore μ has two finite order δ_j Q-open refinements say μ_A and μ_B. Let $\nu = \mu_A \cup \mu_B$. Clearly ν is finite order δ_j Q-open refinements of μ. Hence $A\vee B$ is Q-finitistic.

Remark2.21. Continuous image of Q-Finitistic fuzzy Bitopological space need not be Q-Finitistic. See following example.

Example2.22. Let X be any set and $a\in X$. Let $\delta_1 =\{\underline{0}, \chi_{\{a\}}, \chi_{X-\{a\}}, \underline{1}\}$ and $\delta_2 = \{\underline{0}, \underline{1}\}$. Then clearly both δ_1 and δ_2 are fuzzy topologies on X. Then (X,δ_1,δ_1) and (X,δ_1,δ_2) are fuzzy bitopological spaces. Here (X,δ_1,δ_1) is Q-finitistic but (X,δ_1,δ_2) is not Q-finitistic because $\{\chi_{\{a\}}, \chi_{X-\{a\}}\}$ is δ_2 Q-open cover of X which has no δ_1 finite order Q-open refinement. Let I: $X \to X$ be the identity function. Then I: $(X,\delta_1,\delta_1) \to (X,\delta_1,\delta_2)$ is B-Continuous. It means (X,δ_1,δ_2) is Continuous image of (X,δ_1,δ_1). Here (X,δ_1,δ_1) is Q-Finitistic but (X,δ_1,δ_2) is not Q-Finitistic.

Example2.23. Continuous inverse image of Q-Finitistic fuzzy Bitopological space need not be Q-Finitistic. See following example.

Example2.24. Let X = {a, b}, $\delta_1= \{\underline{0}, \chi_{\{a\}}, \underline{1}\}$ and $\delta_2 = \{\underline{0}, \chi_{\{a\}}, \chi_{\{b\}}, \underline{1}\}$. Then (X,δ_1,δ_2) is Bitopological space and it is not Q-Finitistic. Let Y = {x, y}, $\delta_3 = \{\underline{0}, \chi_{\{x\}}, \underline{1}\}$ and $\delta_4 = \{\underline{0}, \chi_{\{y\}}, \underline{1}\}$. Then (X, δ_3, δ_4) is a fuzzy Bitopological space and it is Q-Finitistic. Define $f: X \to Y$ as f (a) = x and f (b) = y. Then clearly $f: (X,\delta_1, \delta_2) \to (X, \delta_3, \delta_4)$ is continuous. Here (X, δ_3, δ_4) is Q-Finitistic but (X, δ_1, δ_2) which is Q -continuous inverse image of (X, δ_3,δ_4) is not Q**-** Finitistic.

Theorem2.25. Q-finitisticness in fuzzy Bitopological spaces is a topological property.

Question2.26. What about the above results for L- Bitopological spaces.

Answer: If the complete lattice L is also a chain, then all results up to 2.25 hold for L- Bitopological spaces.

Question2.27. What about the results up to 2.24 when L = {0, 1} or {0, ½, 1} or diamond type lattice or P(X). **Answer:** When L = {0, 1} or {0, ½, 1}. Then all results up to 2.25 hold because both {0, 1} and {0, ½, 1} are also chain.

2.27. Exercise: One can check the results for the diamond type lattice L and L =P(X) where P(X) is complete lattice with respect to inclusion relation and order reversing involution as complement of sets. There are several complete lattices for which the results can be checked.

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