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Generating Functions of q-sylvester and q-mittag-leffler Polynomials

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ABSTRACT : The present paper deals with certain generating functions of q-Sylvester, generalized q-Sylvester and q-Mittag-Leffer polynomials.

Keywords.q-Sylvester, generalized q-Sylvester and q-Mittag-Leffer polynomials.
AMS Sub.Classification. 33C45,33C47,33C65

Introduction

The Sylvester polynomials and its generalizations are defined as [12]

$\phi_n(x) = \frac{x^n}{n!}\,{}_2F_0\left(-n, x; -; \frac{-1}{x}\right)$ (1.1) And $f_n(x;a) = \frac{(ax)^n}{n!}\,{}_2F_0\left(-n, x; -; \frac{-1}{ax}\right)$ (1.2) The Mittag-Leffler polynomials are defined as [12] $g_n(x) = 2x\,{}_2F_1(1-n, 1-x; 2; 2)$ (1.3) The aim of the present paper is to define q-analogues of Sylvester, generalized Sylvester and Mittag-Leffler polynomials and give their certain generating functions.

q-SYLVESTER POLYNOMIALS

The q-Sylvester polynomials is denoted by $\Phi_{n,q}(x)$ and is defined as $\Phi_{n,q}(x) = \frac{x^n}{(q;q)_n}\,{}_2\Phi_0\left(q^{-n}, q^x; -; q, \frac{q^n}{x}\right)$ (2.1) which can also be written as

$$\Phi_{n,q}(x) = \sum_{k=0}^{n} \frac{(q^x;q)_k\, x^{n-k}}{(q;q)_k\,(q;q)_{n-k}} \quad (2.2)$$

The following generating functions hold for q-Sylvester polynomials

$\sum_{n=0}^{\infty} \Phi_{n,q}(x) t^n = e_q(xt)\frac{(q^{xt};q)_\infty}{(t;q)_\infty}$ (2.3) and $\sum_{n=0}^{\infty} (q^\lambda;q)_n \Phi_{n,q}(x) t^n = \frac{(q^\lambda xt;q)_\infty}{(xt;q)_\infty}\,{}_2\Phi_1\left(q^\lambda, q^x; q^\lambda xt; q, t\right)$ (2.4)

Generalized q-Sylvester Polynomials

The generalized q-Sylvester polynomials is denoted by $f_{n,q}(x;a)$ and

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is defined as $f_{n,q}(x;a)=\frac{(ax)^n}{(q;q)_n}\,{}_2\Phi_0\left(q^{-n},q^x;-;q,\frac{q^n}{ax}\right)$ (3.1) which can also be written as $f_{n,q}(x;a)=\sum_{k=0}^{n}\frac{(q^x;q)_k\,(ax)^{n-k}}{(q;q)_k\,(q;q)_{n-k}}$ (3.2) where $a\neq 0$ is arbitrary constant.when $a=1$ then (3.1) becomes $f_{n,q}(x;1)=\Phi_{n,q}(x)$(3.3) we call the polynomials $f_{n,q}(x;a)$ generalized q-Sylvester polynomials in view of the relations (3.3).where $\Phi_{n,q}(x)$ is the q-Sylvester polynomials.The following generating functions hold for generalized q-Sylvester polynomials

$\sum_{n=0}^{\infty}f_{n,q}(x;a)t^n=e_q(axt)\frac{(q^x t;q)_\infty}{(t;q)_\infty}$ (3.4) and

$\sum_{n=0}^{\infty}(q^\lambda;q)_n f_{n,q}(x;a)t^n=\frac{(q^\lambda axt;q)_\infty}{(axt;q)_\infty}\,{}_2\Phi_1\left(q^\lambda,q^x;q^\lambda axt;t\right)$(10.3.5)

q-MITTAG-LEFFLER Polynomials:

The q-Mittag-Leffer polynomials is denoted by $g_{n,q}(x)$ and is defined as

$g_{n,q}(x)=-2q[-x]\,{}_2\Phi_1\left(q^{1-n},q^{1-x};q^2;q,2q^n\right)$ (4.1) The following generating functions hold for q-Mittag-Leffer polynomials $1+\sum_{n=1}^{\infty}g_{n,q}(x)t^n={}_1\Phi_1\left(q^{-x};t;q,2t\right)$ (4.2)

PROOF OF (4.2): we have

$$1+\sum_{n=1}^{\infty}g_{n,q}(x)t^n=1+\sum_{n=1}^{\infty}-2q[-x]_2\Phi_1\left(q^{1-n},q^{1-x};q^2;q,2q^n\right)t^n$$

$$=1+\sum_{n=1}^{\infty}-2q[-x]\sum_{k=1}^{n-1}\frac{(q^{1-n};q)_k(q^{1-x};q)_k(2q^n)^k}{(q;q)_k(q^2;q)_k}t^n$$

$$=1-2q[-x]\sum_{n=0}^{\infty}\sum_{k=0}^{n}\frac{(q^{-n};q)_k(q^{1-x};q)_k 2^k q^{kn}(q;q)_n(-1)^k q^{\frac{1}{2}k(k-1)}}{(q;q)_n(q^2;q)_k(q;q)_{n-k}}t^{n+1}=1-2q[-x]t\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\frac{(q;q)_{n+k}(q^{1-x};q)_k 2^k(-1)^k q^{\frac{1}{2}k(k-1)}}{(q;q)_n(q^2;q)_k(q;q)_n}t^{n+k}$$

$$=1-2q[-x]t\sum_{k=0}^{\infty}\frac{(q^{1-x};q)_k(2t)^k(-1)^k q^{\frac{1}{2}k(k-1)}}{(q^2;q)_k}\sum_{n=0}^{\infty}\frac{(q^{1+k};q)_n}{(q;q)_n} \qquad =1-2q[-x]t\sum_{k=0}^{\infty}\frac{(q^{1-x};q)_k(2t)^k(-1)^k q^{\frac{1}{2}k(k-1)}}{(q^2;q)_k}\frac{(q^{1+k}t;q)_\infty}{(t;q)_\infty}$$

$$=1-2q\frac{(1-q^{-x})}{(1-q)}t\sum_{k=0}^{\infty}\frac{(q^{1-x};q)_k(2t)^k(-1)^k q^{\frac{1}{2}k(k-1)}}{(q^2;q)_k(t;q)_{k+1}} \qquad =1+\sum_{k=0}^{\infty}\frac{(q^{-x};q)_{k+1}(2t)^{k+1}(-1)^{k+1} q^{\frac{1}{2}k(k+1)}}{(q^2;q)_{k+1}(t;q)_{k+1}}$$

$$=1+\sum_{n=1}^{\infty}\frac{(q^{-x};q)_n(2t)^n(-1)^n q^{\frac{1}{2}n(n-1)}}{(q^2;q)_n(t;q)_n} \qquad =\sum_{n=0}^{\infty}\frac{(q^{-x};q)_n(2t)^n(-1)^n q^{\frac{1}{2}n(n-1)}}{(q^2;q)_n(t;q)_n}$$

$$={}_1\Phi_1\left(q^{-x};t;q,2t\right) \text{ and}$$

$$\sum_{n=0}^{\infty}g_{n+1,q}(x)\frac{t^n}{(q;q)_n}=-2q[-x]e_q(t)\,{}_1\Phi_1\left(q^{1-x};q^2;q,2t\right) \quad (4.3)$$

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